

NEWFIRM linearity calibration and correction

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Table of Contents

1	Introduction	2
2	Definitions	5
3	The linearity function Λ:	5
4	Deriving Λ from calibration data:	6
5	Applying the linearity correction:	7
6	NEWFIRM array readout timing:	9
7	Linearity calibration observations and analysis	10
8	Results	17
9	Saturation	18
A	Implementation using <code>imexpr</code>	20
A.1	Version 1.1	20
A.2	Version 1.2	21
A.3	Version 1.3	22

Abstract

I discuss the calibration and removal of instrumental nonlinearity for the NEWFIRM array detectors. The analysis is complicated by the time interval between pixel reset and first readout in correlated double sampling observations. This interval can be significant for the NEWFIRM arrays (up to 1.2 seconds for the most common observing mode, and longer when more than one Fowler readout is used), and varies with position over the array. I present a method for deriving linearity correction equations from a sequence of calibration test data, and for applying these corrections to observations. Using a set of calibration observations taken in February 2008, I illustrate the linearity behavior and derive mean linearity coefficients for each of the NEWFIRM arrays. For data taken with low count rates and long exposure times, the arrays are approximately 6% nonlinear at a level of 10000 ADU, and saturate at slightly higher thresholds. In an appendix, I present an implementation of the linearity correction using the IRAF task `imexpr` and expression databases.

Keywords: FEWOMPT

1 Introduction

The NEWFIRM arrays, like most infrared detectors, are nonlinear. For an incident signal with fixed intensity, the counts recorded by the detector increase at a rate that is slower than linear. As more photoelectrons accumulate in a pixel, the rate at which additional electrons accumulate decreases. At count levels of 10000 ADU (or 80000 electrons, given the NEWFIRM gain ~ 8 electrons/ADU), the arrays are typically ~ 6 to 8% nonlinear, and they saturate at a level somewhat higher than this.

It is important to calibrate the nonlinear behavior of infrared arrays, and to remove this when reducing NEWFIRM data, for several reasons:

- to ensure accurate relative photometry between bright sources (e.g., standard stars) and faint sources (e.g., distant galaxies or faint stars of scientific interest),
- to ensure that flat field exposures (dome or sky flats) accurately map the relative pixel responsivity over the array,
- to ensure that the recorded counts from the sky vary in a linear, uniform way over the array as the sky brightness fluctuates, in order to facilitate sky subtraction by running median procedures.

Linearity calibration for NEWFIRM is complicated by the fact that the images which are recorded by the instrument do not include all of the counts that were originally collected in each pixel. The arrays are operated in correlated double sampling (CDS) mode. In CDS, the array is reset, then after a short but finite time interval, each pixel is read once, non-destructively. It takes a significant period of time to read the whole array, and therefore this “reset-to-1st-read” interval varies with pixel position. After the first read, the desired integration time passes, and then the pixels are read again. The final value that is recorded for each pixel is difference between the values of the two readouts. However, the actual, total number of accumulated counts was larger, due to the counts collected during the r2r interval. For the most common broad band observing mode with NEWFIRM (4 digital averages and 1 Fowler sample), the r2r interval varies from 0.0346 to 1.1946 seconds, depending on pixel position on the array. For short exposures, this can be a significant fraction of the total requested exposure time, and for high count rates, the “true” number of counts collected in a pixel may be very different from the “measured” counts recorded after the CDS difference, and therefore may place that pixel on a very different part of the linearity curve. This r2r time is much longer when multiple Fowler sampling readouts are used (e.g., as a strategy to reduce the effective detector readout noise in low-background conditions such as narrow band imaging).

Figure 1 shows a schematic (but realistic) representation of NEWFIRM linearity behavior. Here, the number of counts recorded by the array departs quadratically from the true number of counts that should have been recorded if the array were linear, up to a saturation threshold at 12000 ADU. Figure 1 illustrates the number of counts that would actually be measured in a CDS observation with requested integration time t_i , taking into account the signal from the r2r interval t_r that gets subtracted away. The figure at left shows how the apparent saturation threshold decreases for

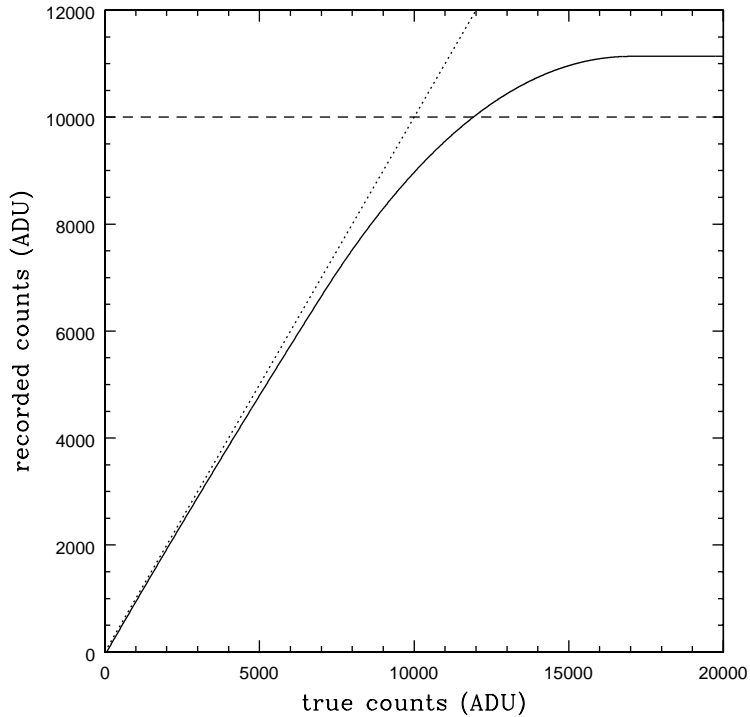


Figure 1: Schematic illustration of infrared array nonlinearity. For a true, linear count rate r_0 incident on the detector, the horizontal axis shows the “true counts” for an exposure time t , i.e., $n = r_0 \times t$, while the vertical axis shows the counts actually collected by the pixel. The dotted line shows a linear relation, while the solid line shows a function which departs quadratically in a manner similar to that of the NEWFIRM arrays, and which then saturates at a maximum value of 12000 ADU. The dashed reference line indicates 10000 ADU, where the NEWFIRM arrays depart from linear behavior by about 6%.

shorter exposure times as the r2r time t_r becomes a significant fraction of the total exposure time $t_t = t_i + t_r$. At right, we see how the variation in the r2r time t_r over the array can lead to different linearity and saturation behavior for a fixed (short) exposure time.

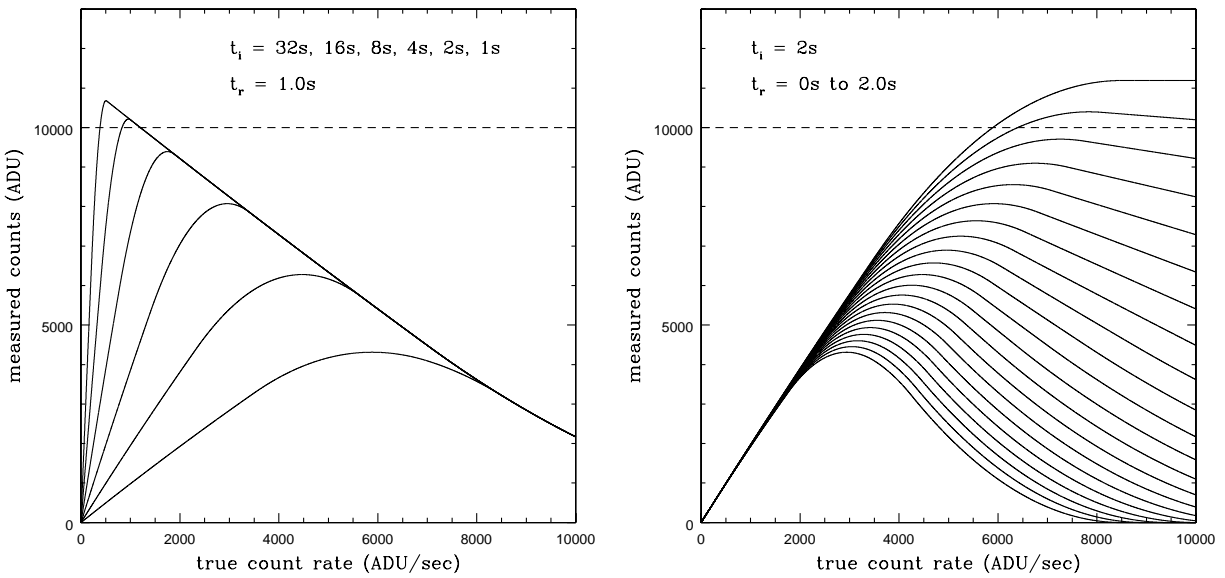


Figure 2: *Left:* Measured counts for schematic NEWFIRM array behavior, as a function of true incident count rate (r_0), for various exposure times $t_i = 1$ to 32 seconds, assuming a reset-to-1st-read interval $t_r = 1.0$ second. *Right:* Measured counts for a short (2 second) exposure as a function of incident count rate, for various reset-to-read intervals from 0 to 2 seconds.

2 Definitions

Here, we define variables and notation that we will use in this discussion.

$$\begin{aligned}
 r_0 &= \text{true incident count rate (if detectors were linear)} \\
 t_r &= \text{reset-to-1st-read (“r2r”) time interval} \\
 t_i &= \text{requested integration time} \\
 t_t &= t_r + t_i
 \end{aligned}$$

We adopt the notation:

$$\begin{aligned}
 N &\equiv \text{measured or collected counts (i.e., actual nonlinear counts)} \\
 n &\equiv \text{“true” linear counts} = rt
 \end{aligned}$$

Specifically:

$$\begin{aligned}
 N_m &= \text{measured counts} \\
 N_r &= \text{counts accumulated in r2r interval} \\
 N_t &= \text{total counts accumulated in pixel} = N_r + N_m
 \end{aligned}$$

Similar definitions apply for n_m, n_r, n_t .

3 The linearity function Λ :

We define the relation between true (linear, n) and measured (nonlinear, N) counts as:

$$N = \Lambda n$$

Λ is often well approximated by a simple linear function of n , up to some count level (N_{sat}) where saturation occurs:

$$N < N_{sat} : \Lambda(n) = b + sn$$

where s is generally negative, and it is common to require $b = 1$, i.e., to define the array to be linear at the point where the counts N (or n) approach 0. Many elements of the discussion here could be generalized to other functional forms, but for simplicity we will assume this form here.

4 Deriving Λ from calibration data:

A linearity calibration sequence generally consists of a series of exposures of a source (e.g., dome flat white spot) with temporally stable and uniform illumination. The illumination level should be set low enough so that a wide range of exposure times can be taken, sampling the dynamic range of accumulated count levels from small values up to saturation (> 10000 ADU for NEWFIRM). Bias+dark exposures must also be taken for each exposure time used in the linearity sequence. Often, frames at a fiducial reference exposure time (some relatively short exposure) are taken periodically during the sequence to monitor possible drifts in the illumination level. However, with NEWFIRM this is complicated by signal persistence, which also tends to affect the counts in the reference images if they are taken after much longer exposures with higher count levels. Frequently, multiple frames will be taken at each exposure time, both for the lights-on data and the bias+dark frames. Section 8 provides a more detailed and specific description of how linearity coefficients were derived from one set of test data taken in February 2008.

Assume that k linearity exposures are taken, with requested exposure times

$$t_l(i), i = 1, k$$

Here, we assume that multiple frames taken at each exposure time have been averaged, and that the appropriate bias+dark exposures have been subtracted. A given pixel (x, y) on the array has reset-to-1st-read interval t_r , and records measured values $N_l(i)$ for the k exposures in the sequence. For the present, we will assume that the illumination level was constant, i.e., not varying with time.

First, we iteratively estimate the “true” linear count rate r_0 by extrapolating the measured count rates back to $t = 0$:

First iteration:

- define the apparent count rate to be $r(i) = N_l(i)/t_l(i)$
- consider only measurements with $N_l(i) < N_{sat}$ to use in the analysis
- carry out linear least-squares fit to $r(i)$ vs. $t_l(i)$: $r(i) = r_0 + at_l$
- record the slope = a (note: this is different than the slope s in the definition of $\Lambda(n)$ because here we fit versus t rather than n)
- the intercept at $t_l = 0$ gives a first estimate of linear count rate r_0 ,

Subsequent iterations:

- include reset-to-read time: $t_t(i) = t_l(i) + t_r$
- estimate counts from reset: $N_r = (r_0 + at_r)t_r$
- add reset counts to measured counts per image: $N_{lr}(i) = N_l(i) + N_r$
- compute new total count rate $r(i) = N_{lr}(i)/t_t(i)$

- carry out linear least-squares fit to $r(i)$ vs. $t_t(i)$ to get new estimates of r_0 and a ,
- repeat iterations a few times until the count rate r_0 converges

With this, we compute

$$\begin{aligned} n(i) &= r_0 t_t(i) \\ N_{lr}(i) &= N_l(i) + (at_r + r_0)t_r \\ r(i) &= N_{lr}(i)/t_t(i) \end{aligned}$$

and do a linear least squares fit to $r(i)$ vs. $n(i)$, for $N < N_{sat}$, to derive the linearity function slope s . This can be done for every pixel on the array, giving a linearity coefficient image $s(x, y)$.

5 Applying the linearity correction:

During a particular observation, a pixel (x, y) observes signal that generates a true (linear, unknown) count rate r_0 . After a requested exposure time t_i , records the (nonlinear, measured) value N_m counts in a correlated double sampling readout. The reset-to-1st-read time for this pixel is t_r , and the measured counts in the CDS readout are true “total” counts recorded, N_t , and the counts collected in the reset-to-read interval, N_r . Again, we notate the true (linear, but unknown) values of the signal as n (i.e., n_m, n_t, n_r), with $N = \Lambda n$.

We have:

$$\begin{aligned} n_t &= r_0 t_t = r(t_r + t_i) \\ n_r &= r_0 t_r \\ N_t &= \Lambda(n_t)n_t \\ N_r &= \Lambda(n_r)n_r \\ N_m &= N_t - N_r \\ &= r_0[t_t \Lambda(n_t) - t_r \Lambda(n_r)] \\ &= r_0[t_t \Lambda(r_0 t_t) - t_r \Lambda(r_0 t_r)] \end{aligned}$$

Therefore, for any specified linearity function $\Lambda(n)$, and for fixed t_r and t_i , there is a unique mapping between “true” linear count rate r_0 and the measured counts N_m . In practice this mapping can be bimodal, in the sense that different count rates r_0 may map to the same measured N_m .

Once again, let’s adopt a model linearity function of the form:

$$\Lambda(n) = b + sn$$

where s is generally negative. As noted above, we may wish to define $\Lambda(n)$ such that $b = 1$, but for the moment we will carry along the variable b explicitly.

So, we have:

$$\begin{aligned} N &= \Lambda(n)n \\ &= n(b + sn) \end{aligned}$$

i.e., a quadratic relation between true and measured counts. From above:

$$\begin{aligned} N_m &= r_0[t_t\Lambda(rt_t) - t_r\Lambda(r_0t_r)] \\ &= r_0[t_t(b + sr_0t_t) - t_r(b + sr_0t_r)] \\ &= r_0[b(t_t - t_r) + sr_0(t_t^2 - t_r^2)] \\ &= bt_t r_0 + s(t_t^2 - t_r^2)r_0^2 \end{aligned}$$

Solving this quadratic equation gives:

$$r_0 = [-bt_i \pm \sqrt{(bt_i)^2 + 4s(t_t^2 - t_r^2)N_m}] / (2s(t_t^2 - t_r^2))$$

The “-” solution represents the “descending” branch of the $N_m(r)$ relation, where the final readout is highly saturated and the signal in the first readout is catching up to it. In practice those data are unrecoverable, and only the “+” solution is valid, so:

$$r_0 = [-bt_i + \sqrt{(bt_i)^2 + 4s(t_t^2 - t_r^2)N_m}] / (2s(t_t^2 - t_r^2))$$

For the case where we have defined $b = 1$, this gives:

$$r_0 = [-t_i + \sqrt{(t_i)^2 + 4s(t_t^2 - t_r^2)N_m}] / (2s(t_t^2 - t_r^2))$$

Generally, both the reset-to-1st-read time t_r and the linearity coefficient s may be functions of array position (x, y) .

Finally, if one desires the linearized counts for the final product (instead of the count rate), we simply multiply by the exposure time:

$$n_m = r_0 t_i$$

6 NEWFIRM array readout timing:

The discussion on the readout timing of the NEWFIRM arrays is based on information provided by Maureen Ellis, based on timing tests that she carried out.

The interval between array reset and the readout of the first pixel is $t_0 = 0.0346$ seconds. One CDS readout (2048 rows) requires a time (t_{cds}) which depends on the number of digital averages (N_{DA}):

N_{DA}	t_{cds}
1	0.56
2	0.86
4	1.16
8	2.75
16	2.94

Note that for $N_{DA} \geq 2$, the timing can be represented by the linear relation

$$t_{cds} = 0.564 + 0.148N_{DA}$$

but that this apparently does not hold for $N_{DA} = 1$. After discussions with Maureen Ellis, it is not entirely clear why the timing behavior should depart from this linear relation for $N_{DA} = 1$. It may be worthwhile to redo these measurements at some point to confirm this behavior.

The total r2r time interval is:

$$t_r(y) = t_0 + t_{cds}(y/2048)$$

For the case of readout using a single Fowler sample ($N_{FS} = 1$), the equation above gives the reset time vs. pixel position y . For multiple Fowler samples ($N_{FS} > 1$), the array is read out N_{FS} times in succession. The first readout takes place as described above. Each subsequent readout follows after an inter-readout interval $t_1 = 0.0568$ sec. Therefore, for Fowler readouts $i = 1$ to N_{FS} , a given pixel will be read at the following times:

$$t_r(y, i) = t_0 + t_{cds}(y/2048) + (i - 1)(t_{cds} + t_1); i = 1, N_{FS}$$

The multiple Fowler sample readouts are summed and the image is written out. For linearity calibration, we will adopt the average $\langle t_r(y) \rangle$ for the N_{FS} Fowler samples as the appropriate reset-to-1st-read interval:

$$\langle t_r(y) \rangle = t_0 + (t_{cds} + t_1)(N_{FS} - 1)/2 + t_{cds}(y/2048)$$

Note: this expression works fine for $N_{FS} = 1$, since the second term then vanishes and the expression is reduced to the same as that used for $N_{FS} = 1$ (above). So, we can use this in the general case. We will write this as:

$$\begin{aligned}
t_r(y) &= t_{r0} + t_{cds}(y/2048) \\
t_{r0} &= t_0 + (t_{cds} + t_1)(N_{FS} - 1)/2
\end{aligned}$$

where t_{r0} is a function of both N_{DA} and N_{FS} , and t_{cds} is a function of N_{DA} . The table below gives these values for various combinations of N_{DA} and N_{FS} . The last column gives the sum $t_0 + t_{cds}$, which is the maximum “extra” exposure time that is accumulated during the reset-to-1st-readout interval (at readout row $y = 2048$). When considering saturation (e.g., for flat fields, sky brightness, standard stars, etc.), an observer should keep in mind that some portions of the array will effectively have this much “extra” unseen exposure time in addition to whatever exposure time that they requested, and that this extra signal will affect linearity and saturation.

N_{DA}	N_{FS}	t_{cds}	t_{r0}	$t_0 + t_{cds}$
1	1	0.56	0.0346	0.595
2	1	0.86	0.0346	0.894
4	1	1.16	0.0346	1.195
8	1	1.75	0.0346	1.785
16	1	2.93	0.0346	2.965
1	4	0.56	0.960	1.520
2	4	0.86	1.410	2.270
4	4	1.16	1.854	3.014
8	4	1.75	2.742	4.492
16	4	2.93	4.518	7.448
1	8	0.56	2.193	2.753
2	8	0.86	3.243	4.103
4	8	1.16	4.279	5.439
8	8	1.75	6.351	8.101
16	8	2.93	10.495	13.425

7 Linearity calibration observations and analysis

Linearity calibration data were taken on a number of occasions in the 2007B and 2008A semesters. Here, we concentrate on the most useful sequence of observations, taken by Ron Probst on 2008 February 13.

Figure 3 illustrates the layout of the four NEWFIRM arrays, their orientation with respect to the sky, and their storage within multiextension FITS files, as implemented for the 2008A observing semester. (These orientations were different for observations taken in 2007A and 2007B.) In the figures that follow, the four arrays are indicated by number labels [1] to [4] which indicate the FITS extension from the 2008A data.

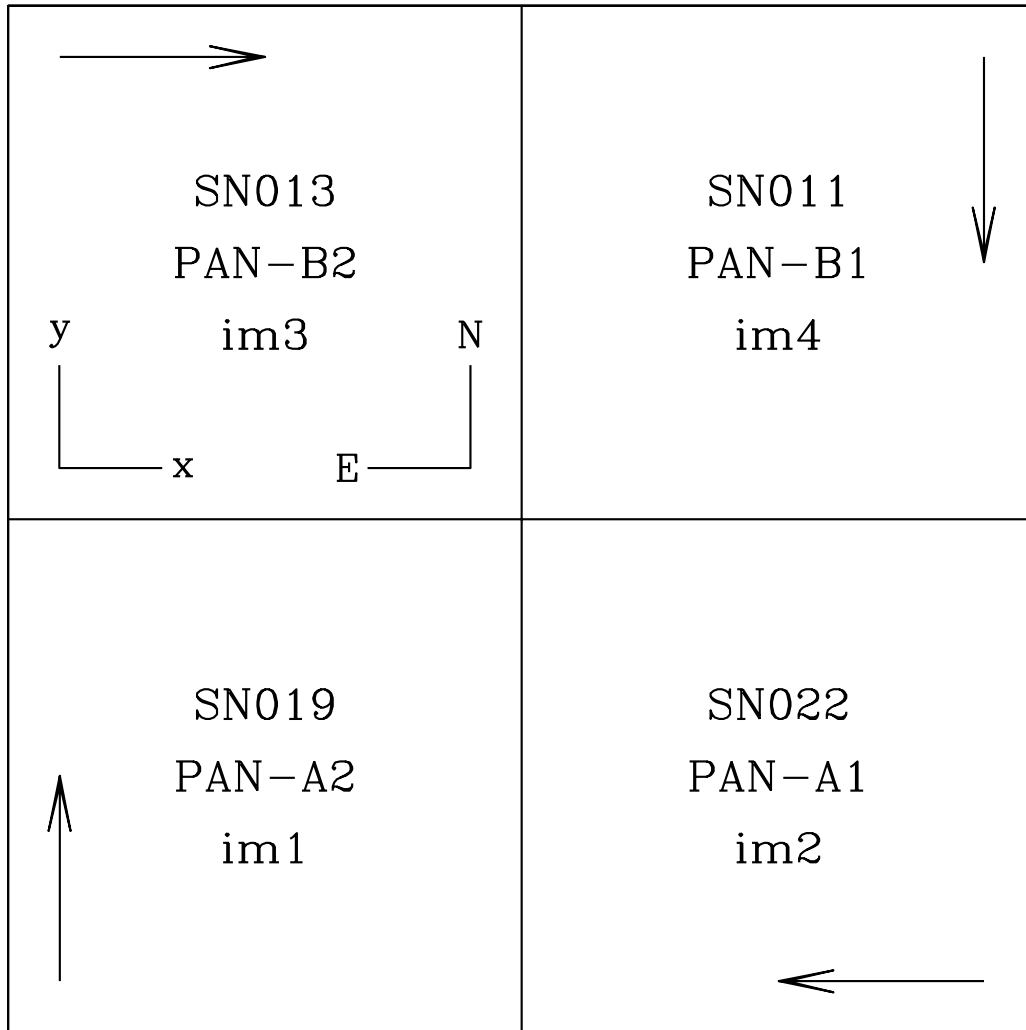


Figure 3: NEWFIRM array orientations and FITS file layout for 2008A semester. Labels in each quadrant indicate the detector identifier (e.g., SN019), the pixel acquisition node (PAN, e.g., PAN-A2) used to read out that array, and the FITS image extension (e.g., im2) in which data from that array is recorded. Arrows in each quadrant indicate the readout direction within each amplifier. In the upper left hand quadrant, a compass rose indicates the orientation of the celestial equatorial coordinate system, while another shows the orientation of the $+x$, $+y$ pixel axes. These are the same for all four quadrants.

The linearity calibration test data were taken using dome flats as the source of illumination, with the lamp intensity set at 20.5 V. Observing through the J filter, this produces count rates of order 200 to 270 ADU/s, depending on the detector. (The SN019 array has extra anti-reflection coating, which reduces the count rate at shorter wavelengths relative to those of the other three detectors.) A series of exposures was taken through the J filter, and then repeated with the dark slide in. The exposure time was gradually increased throughout the sequence, from the shortest allowed exposures (1.2 seconds) up to 60 seconds, with frequent returns to a 10 second reference time. 4 to 6 exposures were taken at each exposure time. The full observing sequence was:

1. 6 x 10.0s reference	13. 4 x 25.0s	25. 4 x 45.0s
2. 6 x 1.213s	14. 6 x 10.0s reference	26. 6 x 10.0s reference
3. 6 x 2.5s	15. 4 x 27.5s	27. 4 x 47.5s
4. 6 x 5.0s	16. 4 x 30.0s	28. 4 x 50.0s
5. 6 x 7.5s	17. 6 x 10.0s reference	29. 6 x 10.0s reference
6. 6 x 10.0s reference	18. 4 x 32.5s	30. 4 x 52.5s
7. 6 x 12.5s	19. 4 x 35.0s	31. 4 x 55.0s
8. 6 x 15.0s	20. 6 x 10.0s reference	32. 6 x 10.0s reference
9. 6 x 17.5s	21. 4 x 37.5s	33. 4 x 57.5s
10. 6 x 10.0s reference	22. 4 x 40.0s	34. 4 x 60.0s
11. 4 x 20.0s	23. 6 x 10.0s reference	35. 6 x 10.0s reference
12. 4 x 22.5s	24. 4 x 42.5s	

During analysis, multiple frames taken at each exposure time were averaged with sigma-clipping to remove outlying pixel values. Figure 4 shows the count levels (N , in ADU), averaged over each of the four arrays, versus exposure time. The counts increase up to a threshold above than 10000 ADU (somewhat different for each array), then flatten out as the array reaches saturation. Figure 5 shows the apparent count rate (N/t) versus the measured counts N . This illustrates the mild nonlinearity as N increases to about 10000 ADU, and the sharp rollover beyond that.

Figure 6 illustrates how linearity coefficients are derived from these data, following the iterative procedure described in Section 4. The “true” linear count rate r_0 was estimated by extrapolating the measured counts versus exposure time down to $t = 0$, and the procedure was iterated to take into account the “missing” signal from the reset-to-1st-readout interval. The 2008 February 13 linearity sequence used 4 digital averages ($N_{DA} = 4$) and single Fowler sampling ($N_{FS} = 1$). The average reset-to-1st-read interval over the array is then $\langle t_r \rangle = 0.0346 + 1.16/2 = 0.6146$ seconds, and the analysis done here (which derives an average coefficient over the whole array) adopts this constant value. This approximation has little effect, since the actual exposure times used in the linearity sequence go up to $\sim 100\times$ longer than the r2r interval, but a careful derivation of linearity coefficients for each pixel could use the correct r2r interval for each pixel.

The mean linearity coefficients per array, derived from the 2008A calibration data using this method are:

Array	Extension	Coefficient
SN019	im1	-5.404e-6
SN022	im2	-5.952e-6
SN013	im3	-6.123e-6
SN011	im4	-7.037e-6

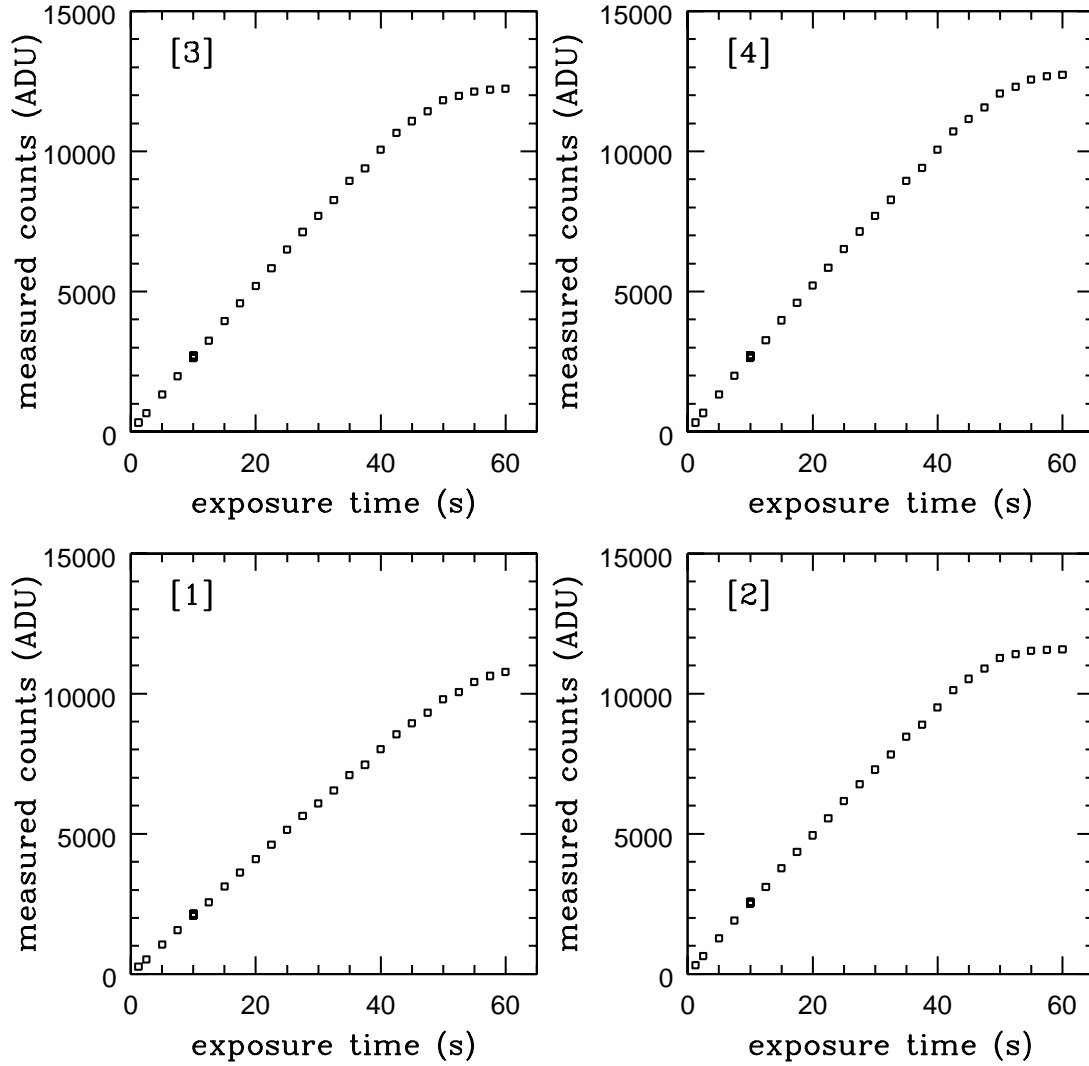


Figure 4: Average counts versus exposure time from linearity calibration observing sequence.

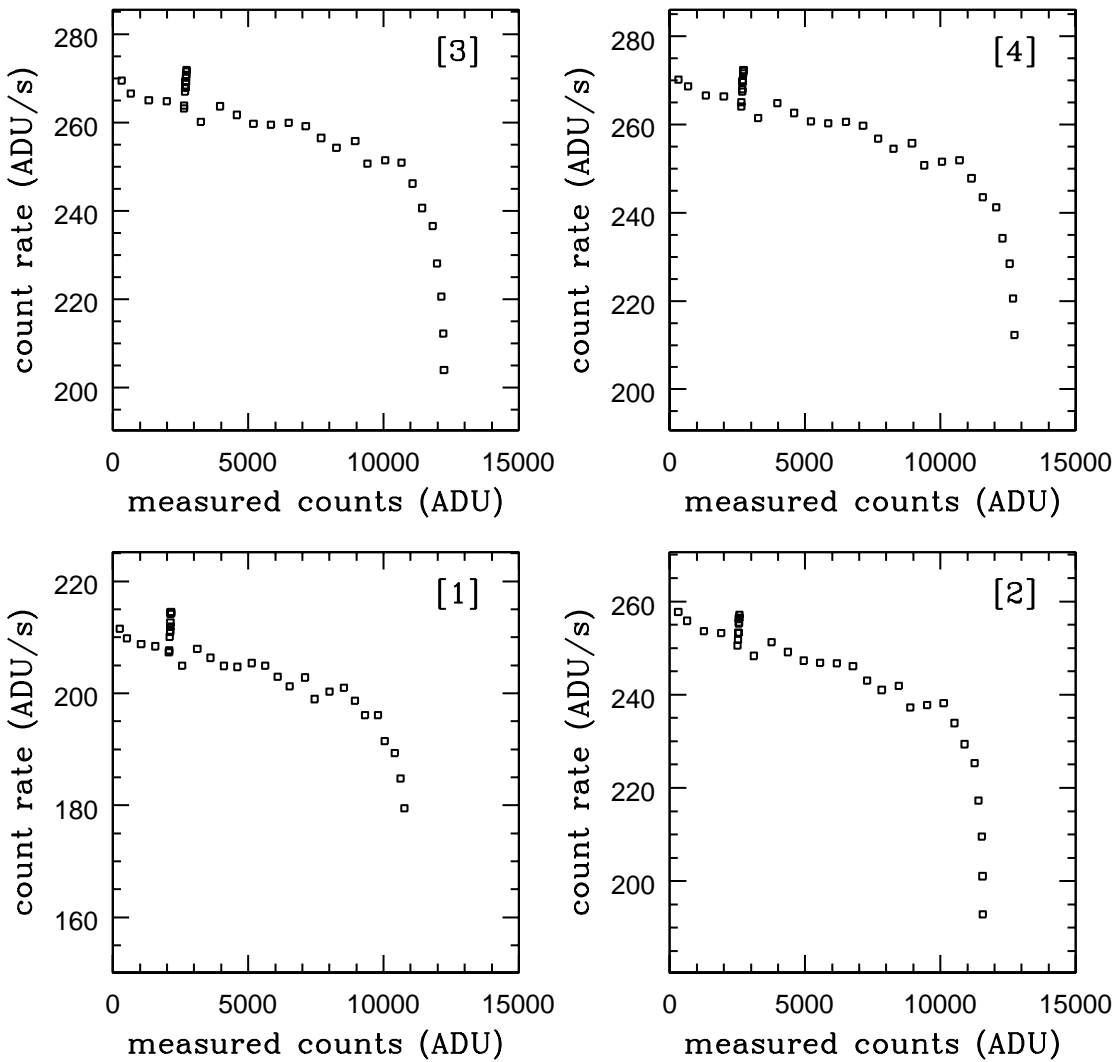


Figure 5: Apparent count rate versus measured counts from linearity calibration observing sequence. The bunch of points at similar (low) ADU levels are the various 10 second reference frames taken during the sequence.

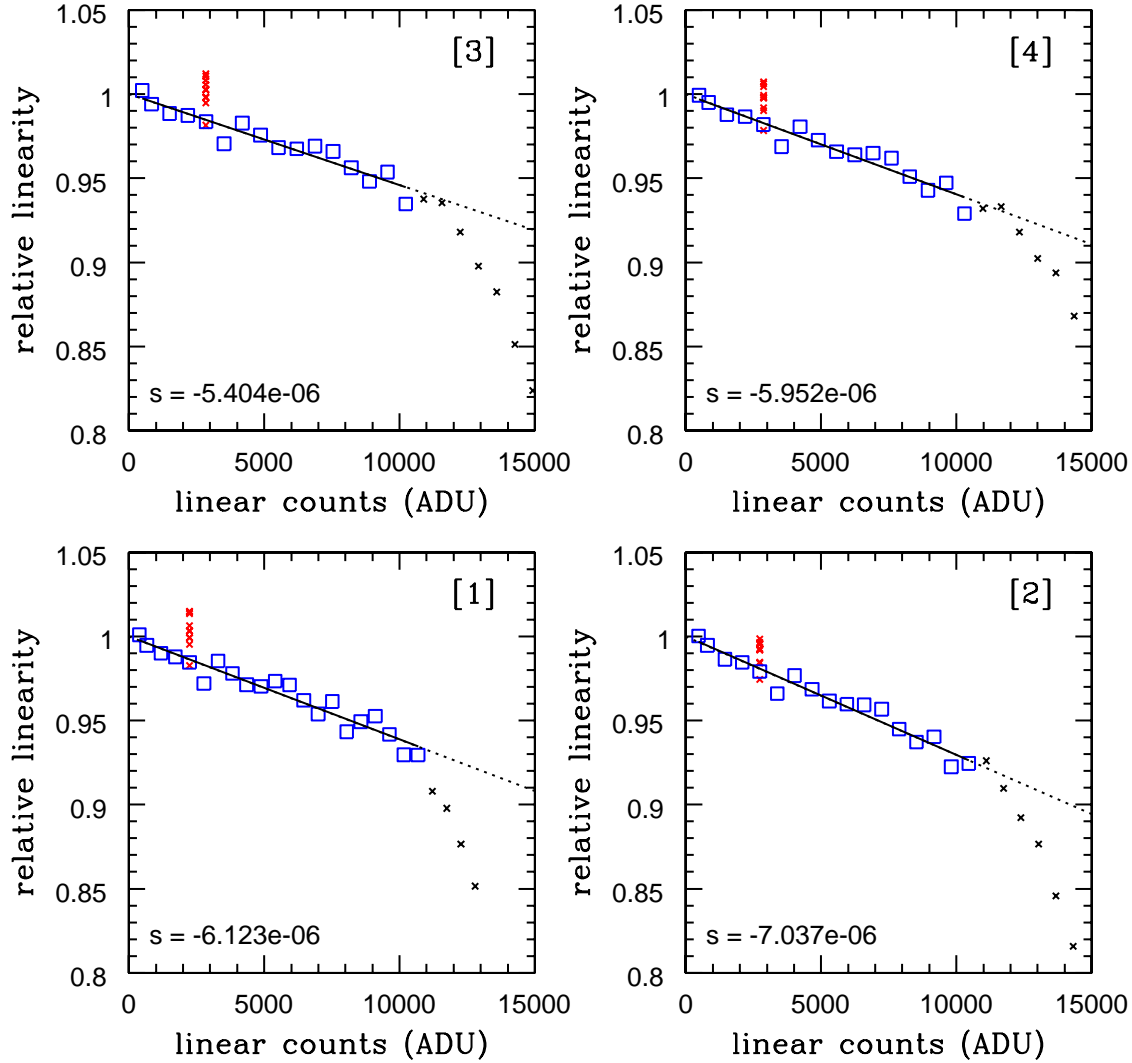


Figure 6: Relative linearity (Λ) versus true, linear counts ($n_t = r_0 t_t$). The red x's are the 10s reference frames, most of which were not used in the fitting. The black x's are data with count levels $N > 10000$ ADU, also not used. The blue boxes are the points for which the linearity relation was fit using the functional form $\Lambda(n) = 1 + sn$, and the value of the coefficient s is indicated in each panel.

8 Results

The following table shows the mean amplitude of the linearity correction, and the minimum to maximum range of the correction values, for idealized images with uniform count levels of 1000, 5000, 7000, and 9000 ADU (recorded, nonlinear values, as if from a CDS exposure), with exposure times from 1.25 seconds to 20 seconds. These were calculated using a fixed linearity coefficient with a value $s = -6 \times 10^{-6}$, which is typical for the NEWFIRM arrays. The simulation assumes the array timings for $N_{DA} = 4$ and $N_{FS} = 1$, which are the recommended values for standard broad band imaging. The last column shows the percentage of pixels for which the “true” (i.e., linear) number of counts, including the signal collected during the readout-to-1st-read interval, exceeded 10000 ADU. Italics indicate images where some pixels exceeded 12000 ADU, i.e., pushing well into the range of likely saturation.

Measured counts (ADU)	Exposure time (sec)	Average correction (%)	Max/Min range (%)	>10000 ADU (%)
1000	1.25	1.20%	1.19%	0%
1000	2.50	0.90%	0.60%	0%
1000	5.00	0.80%	0.30%	0%
1000	10.00	0.70%	0.20%	0%
1000	20.00	0.60%	0.10%	0%
5000	1.25	6.86%	7.06%	13.7%
5000	2.50	4.94%	3.25%	0%
5000	5.00	4.04%	1.59%	0%
5000	10.00	3.62%	0.78%	0%
5000	20.00	3.40%	0.39%	0%
7000	1.25	10.30%	11.15%	68.2%
7000	2.50	7.23%	4.92%	33.4%
7000	5.00	5.87%	2.33%	0%
7000	10.00	5.22%	1.15%	0%
7000	20.00	4.90%	0.56%	0%
9000	1.25	14.34%	16.66%	98.5%
9000	2.50	9.77%	6.91%	94.0%
9000	5.00	7.83%	3.19%	85.0%
9000	10.00	6.93%	1.55%	66.9%
9000	20.00	6.50%	0.76%	30.8%

While this example is highly idealized, it illustrates several important points. When exposure times are short, the reset-to-1st-readout interval becomes significant compared to the requested exposure time. In this circumstance, when the count rates are high, the linearity corrections become quite large and vary significantly over the array due to the timing variations. Also, a significant number of pixels can be pushed into the range of nominal saturation. This is true even if the *apparent* number of counts in the image is well below 10000 ADU.

9 Saturation

As Figure 1 shows, no definition of saturation in terms of a fixed threshold number of counts can be used for NEWFIRM data. The counts in a CDS readout will saturate at different apparent levels, depending on the intrinsic count rate, the exposure time, and the reset-to-1st-read time interval.

However, we may usefully define a saturation threshold in terms of the true number of counts that would have been incident on a pixel if the detector behavior were linear, including both the integration time and the reset-to-1st-read time. Let us call this n_s . For NEWFIRM, a value $n_s = 10000$ ADU should be a reasonable and conservative choice. Note that unlike the schematic horizontal dashed line shown in Figure 1, which defines saturation in terms of *measured* (nonlinear) counts, here instead we are defining this threshold in terms of *true* (linear) counts – i.e., as a vertical line in those figures.

With both t_i and t_r known for each pixel, as well as the linearity function $\Lambda(n)$ (here assumed to be $\Lambda(n) = 1 + sn$), we may compute the relation between the true saturation level n_s and the equivalent measured (nonlinear) value, N_s , i.e., essentially the rollover point in the curves shown in Figure 1. Going back to the equations from § 5:

$$\begin{aligned} N_m &= N_t - N_r \\ &= n_t \Lambda(n_t) - n_r \Lambda(n_r) \\ &= n_t(1 + sn_t) - n_r(1 + sn_r) \end{aligned}$$

$$\begin{aligned} r_0 &= \frac{n_t}{t_t} = \frac{n_t}{t_r + t_i} \\ n_r &= r_0 t_r = n_t \left(\frac{t_r}{t_r + t_i} \right) \end{aligned}$$

Rearranging some terms in these equations, and substituting the “true” saturation value n_s for n_t , we get the “measured” saturation value N_s : we get:

$$N_s = n_s \left[1 - \frac{t_r}{t_r + t_i} \right] + sn_s^2 \left[1 - \left(\frac{t_r}{t_r + t_i} \right)^2 \right]$$

It is important to understand that this approach only defines a threshold value for saturation as the signal brightness (and hence count rate) increases, i.e., moving from left to right along the horizontal axis of Figure 1. It *cannot* recognize pixels whose true count rates are so high that the measured counts N_m have turned over and started to decrease again (see Figure 1). This happens when the data have truly saturated to the maximum value well before the final CDS readout, and when a significant number of counts are accumulated during the first CDS readout and subtracted away. These saturated pixels will have values below the nominal saturation threshold N_s defined

above, and cannot easily be detected directly. Other strategies may need to be employed. For the case of astronomical sources (e.g., stars), we may hope to rely on the fact that most objects are centrally peaked, leading to a “volcano effect”, in which the pixel count values rise toward a maximum value somewhere near our threshold N_s , then turn over and decrease in a “crater” at the center of the star. By detecting “rings” of saturated pixels, we may also flag those lower-valued but still saturated pixels that they surround. A more difficult case is the example of a fully saturated sky or flatfield image. This may be very difficult to recognize by any simple thresholding method.

A Implementation using `imexpr`

Within the IRAF framework, NEWFIRM linearity correction can be implemented using the `imexpr` task. The `imexpr.exprdb` parameter provides a mechanism by which fairly complex mathematical expressions can be specified in a database file and then called by `imexpr`.

Here, we sketch three implementations of linearity correction using `imexpr` databases. These put increasing sophistication within the database expressions, at the cost of increased computation time due to the fact that `imexpr` is required to recompute constants and evaluate conditional expressions repeatedly, thousands of times per image.

Database version 1.1 has the simplest expressions. The timing constants t_{cds} and t_0 (see §6) should be computed once for the image being evaluated and then passed to `imexpr` as arguments. Different functions (`lc1`, `lc2`, `lc3`, `lc4`) are called for the four different NEWFIRM detectors to take into account their different readout orientations within the FITS files. Version 1.2 adds a bit of complexity (and inefficiency) by calculating t_{cds} and t_0 within the expression database. Version 1.3 is fully self-contained, and uses a single function `lc` to correct all four detectors, conditionally selecting which of several sub-functions to call based on header information that identifies the detector. In a simplified timing test using a single constant linearity coefficient rather than an array of coefficients, and processing on a lightly loaded Macintosh Pro workstation, the mean times needed to apply a correction to a single 2048^2 image for these three versions were 4.4, 5.2, and 9.5 seconds, respectively. The added sophistication of v1.3 may therefore be outweighed by the time savings that result from using one of the less sophisticated implementations.

A.1 Version 1.1

```
# Database of imexpr function information for NEWFIRM linearity correction
# Draft 1.1, MED, 2008 September 2
#
# Apply linearity corrections to any NEWFIRM detector.
# This version uses the fewest 'expandable' expressions within the imexpr database,
# reducing computation time by evaluating the parameters tcds and tr0 once in the
# calling routine. It employs four distinct functions, 'lc?', which are operate on
# the four NEWFIRM detectors.
#
# For the four NEWFIRM detectors (in extensions 1,2,3,4), the format of the imexpr call is (e.g.):
# imexpr "lc1(a,b)" out1.fits a=image.fits[1] b=lincoeff.fits[1] c=tcds d=tr0 exprdb=newfirm_linearity_v1.1.db
# imexpr "lc2(a,b)" out2.fits a=image.fits[2] b=lincoeff.fits[2] c=tcds d=tr0 exprdb=newfirm_linearity_v1.1.db
# imexpr "lc3(a,b)" out3.fits a=image.fits[3] b=lincoeff.fits[3] c=tcds d=tr0 exprdb=newfirm_linearity_v1.1.db
# imexpr "lc4(a,b)" out4.fits a=image.fits[4] b=lincoeff.fits[4] c=tcds d=tr0 exprdb=newfirm_linearity_v1.1.db
#
# where (e.g.)
# out1.fits          is the result of the operation, a linearized version of the image,
# i.e., where the original signal level has been corrected for nonlinearity.
# a = image.fits[1]  is the image being corrected (in this example, one extension of an MEF)
# b = lincoeff.fits[1] is either an array of linearity coefficients (again, in this example,
# one extension of an MEF), or a single constant coefficient to be used
# for all pixels in the array
# c = tcds           is the time required for a single readout of the detector in correlated double
# sampling (CDS) mode. This is a function of the number of digital averages (NDA):
# tcds = ((NDA==1) ? 0.56 : 0.564+0.148*NDA)
# d = tr0            is the baseline time constant for reset-to-1st-read. This is a function
# if the single CDS readout time (tcds) and the number of Fowler Samples (NFS),
# tr0 = (TR0+(tcds+TR1)*(NFS-1)/2),
# where
# TR0 = 0.0346 is the interval (sec) between detector reset and first pixel readout
# TR1 = 0.0568 is the minimum interval (sec) between end of 1 CDS readout and start of another

# Timing constants:
TX= a.exptime # Requested exposure time (from image header EXPTIME keyword)
```

```

# Functions to compute the reset-to-1st-readout time interval as a function of pixel position.
# The four functions 'tr?' are used for the four NEWFIRM arrays, and take the detector readout
# orientations into account, based on the array configuration and FITS format used in 2008A.
# Note that this will be different for 2007B data, where the arrangement of the detectors
# in the FITS images was different.

tr1(c,d) (d+c*(J/2048.)) # For SN019 = im1, SE quadrant
tr2(c,d) (d+c*(2049.-I)/2048.) # For SN022 = im2, SW quadrant
tr3(c,d) (d+c*(I/2048.)) # For SN013 = im3, NE quadrant
tr4(c,d) (d+c*(2049.-J)/2048.) # For SN011 = im4, NW quadrant

# Linearity correction functions for the four NEWFIRM detectors. The calling imexpr routine
# would call one of these four functions (which in turn call the appropriate 'tr?' function)
# depending on the array being processed.

lc1(a,b) (TX*(sqrt(TX**2+4.*a*b*((TX+tr1(c,d))**2-tr1(c,d)**2))-TX)/(2.*b*((TX+tr1(c,d))**2-tr1(c,d)**2))
lc2(a,b) (TX*(sqrt(TX**2+4.*a*b*((TX+tr2(c,d))**2-tr2(c,d)**2))-TX)/(2.*b*((TX+tr2(c,d))**2-tr2(c,d)**2))
lc3(a,b) (TX*(sqrt(TX**2+4.*a*b*((TX+tr3(c,d))**2-tr3(c,d)**2))-TX)/(2.*b*((TX+tr3(c,d))**2-tr3(c,d)**2))
lc4(a,b) (TX*(sqrt(TX**2+4.*a*b*((TX+tr4(c,d))**2-tr4(c,d)**2))-TX)/(2.*b*((TX+tr4(c,d))**2-tr4(c,d)**2))

```

A.2 Version 1.2

```

# Database of imexpr function information for NEWFIRM linearity correction
# Draft 1.2, MED, 2008 September 2
#
# Apply linearity corrections to any NEWFIRM detector.
# This version uses four functions, 'lc?', which are operate on the four NEWFIRM detectors.
# It also includes the evaluation of the tcds and tr0 functions within the imexpr database.
# This simplifies the call and keeps various constants and array-dependent timing calculations
# within the database, rather than hardwiring them in the calling routine, but comes at the
# cost of slowing down execution somewhat.
#
# For the four NEWFIRM detectors (in extensions 1,2,3,4), the format of the imexpr call is (e.g.):
# imexpr "lc1(a,b)" out1.fits a=image.fits[1] b=lincoeff.fits[1] exprdb=newfirm_linearity_v1.2.db
# imexpr "lc2(a,b)" out2.fits a=image.fits[2] b=lincoeff.fits[2] exprdb=newfirm_linearity_v1.2.db
# imexpr "lc3(a,b)" out3.fits a=image.fits[3] b=lincoeff.fits[3] exprdb=newfirm_linearity_v1.2.db
# imexpr "lc4(a,b)" out4.fits a=image.fits[4] b=lincoeff.fits[4] exprdb=newfirm_linearity_v1.2.db
#
# where (e.g.)
# out1.fits is the result of the operation, a linearized version of the image,
# i.e., where the original signal level has been corrected for nonlinearity.
# a = image.fits[1] is the image being corrected (in this example, one extension of an MEF)
# b = lincoeff.fits[1] is either an array of linearity coefficients (again, in this example,
# one extension of an MEF), or a single constant coefficient to be used
# for all pixels in the array

# Timing constants:

TR0= 0.0346 # Interval (seconds) between detector reset and first pixel readout
TR1= 0.0568 # Minimum interval (seconds) between end of 1 CDS readout and start of another
TX= a.exptime # Requested exposure time (from image header EXPTIME keyword)
NFS= a.fsamples # Number of Fowler Samples (from image header FSAMPLE keyword)
NDA= a.digavg # Number of Digital Averages (from image header DIGAVGS keyword)

# Reset-to-1st-read time functions for four NEWFIRM detectors.

# Time for a single readout of the detector in correlated double sampling (CDS) mode.
# This is a function of the number of digital averages (NDA).

tcds ((NDA==1) ? 0.56 : 0.564+0.148*NDA)

# Baseline time constant for reset-to-1st-read, as a function of
# the single CDS read time (tcds) and the number of Fowler Samples (NFS):

tr0 (TR0+(tcds+TR1)*(NFS-1)/2)

# Functions to compute the reset-to-1st-readout time interval as a function of pixel position.
# The four functions 'tr?' are used for the four NEWFIRM arrays, and take the detector readout
# orientations into account, based on the array configuration and FITS format used in 2008A.
# Note that this will be different for 2007B data, where the arrangement of the detectors
# in the FITS images was different.

tr1 (tr0+tcds*(J/2048.)) # For SN019 = im1, SE quadrant
tr2 (tr0+tcds*(2049.-I)/2048.) # For SN022 = im2, SW quadrant
tr3 (tr0+tcds*(I/2048.)) # For SN013 = im3, NE quadrant
tr4 (tr0+tcds*(2049.-J)/2048.) # For SN011 = im4, NW quadrant

# Linearity correction functions for the four NEWFIRM detectors. The calling imexpr routine
# would call one of these four functions (which in turn call the appropriate 'tr?' function)
# depending on the array being processed.

```

```
lc1(a,b) (TX*(sqrt(TX**2+4.*a*b*((TX+tr1)**2-tr1**2))-TX)/(2.*b*((TX+tr1)**2-tr1**2)))
lc2(a,b) (TX*(sqrt(TX**2+4.*a*b*((TX+tr2)**2-tr2**2))-TX)/(2.*b*((TX+tr2)**2-tr2**2)))
lc3(a,b) (TX*(sqrt(TX**2+4.*a*b*((TX+tr3)**2-tr3**2))-TX)/(2.*b*((TX+tr3)**2-tr3**2)))
lc4(a,b) (TX*(sqrt(TX**2+4.*a*b*((TX+tr4)**2-tr4**2))-TX)/(2.*b*((TX+tr4)**2-tr4**2)))
```

A.3 Version 1.3

```
# Database of imexpr function information for NEWFIRM linearity correction
# Draft 1.3, MED, 2008 September 2
#
# Apply linearity corrections to any NEWFIRM detector.
# This version uses a single function 'lc' which selects selecting appropriate
# extension-specific functions to call based on the IMAGEID header keyword.
# The calculations here almost fully "self-contained" within the database expressions,
# i.e., the various reset and readout time constants are evaluated with functions within
# the database, and do not need to be specified in the calling function. This is elegant,
# but also means that the calculation of these constants, as well as the evaluation of
# several conditional expressions, is repeated thousands of times, slowing the speed
# of execution.
#
# The format of the imexpr call is (e.g.):
# imexpr "lc(a,b)" out.fits a=image.fits[1] b=lincoeff.fits[1] exprdb=newfirm_linearity_v1.3.db
#
# where
# out.fits is the result of the operation, a linearized version of the image,
# i.e., where the original signal level has been corrected for nonlinearity.
# a = image.fits is the image being corrected (in this example, one extension of an MEF)
# b = lincoeff.fits is either an array of linearity coefficients (again, in this example,
# one extension of an MEF), or a single constant coefficient to be used
# for all pixels in the array

# Timing constants:

TR0= 0.0346 # Interval (seconds) between detector reset and first pixel readout
TR1= 0.0568 # Minimum interval (seconds) between end of 1 CDS readout and start of another
TX= a.exptime # Requested exposure time (from image header EXPTIME keyword)
NFS= a.fsamples # Number of Fowler Samples (from image header FSAMPLE keyword)
NDA= a.digavg # Number of Digital Averages (from image header DIGAVGS keyword)

# The IMAGEID header keyword identifies the original image extension
# and hence specifies which NEWFIRM detector produced the data.

IID= a.imageid

# Reset-to-1st-read time functions for four NEWFIRM detectors.

# Time for a single readout of the detector in correlated double sampling (CDS) mode.
# This is a function of the number of digital averages (NDA).

tcds ((NDA==1) ? 0.56 : 0.564+0.148*NDA)

# Baseline time constant for reset-to-1st-read, as a function of
# the single CDS read time (tcds) and the number of Fowler Samples (NFS):

tr0 (TR0+(tcds+TR1)*(NFS-1)/2)

# Function to compute the reset-to-1st-readout time interval as a function of pixel position.
# The routine 'tr' uses the IMAGEID header keyword to select one of four different functions
# (tr?, below) that operate on the four different NEWFIRM detectors, taking their
# orientations into account.

tr ((IID==1) ? tr1 : ((IID==2) ? tr2 : ((IID==3) ? tr3 : ((IID==4) ? tr4 : trx)))

# The four functions 'tr?' are used for the four NEWFIRM arrays, and take the detector readout
# orientations into account, based on the array configuration and FITS format used in 2008A.
# Note that this will be different for 2007B data, where the arrangement of the detectors
# in the FITS images was different.

tr1 (tr0+tcds*(J/2048.)) # For SN019 = im1, SE quadrant
tr2 (tr0+tcds*(2049.-I)/2048.) # For SN022 = im2, SW quadrant
tr3 (tr0+tcds*(I/2048.)) # For SN013 = im3, NE quadrant
tr4 (tr0+tcds*(2049.-J)/2048.) # For SN011 = im4, NW quadrant
trx (0) # Dummy function if the value of a.imageid is invalid
# No error warning will occur, which may be a problem.

# Generalized Linearity correction functions for all four NEWFIRM detectors
# (calling the generic "tr" function, which selects "tr?" functions by IMAGEID

lc(a,b) (TX*(sqrt(TX**2+4.*a*b*((TX+tr)**2-tr**2))-TX)/(2.*b*((TX+tr)**2-tr**2)))
```